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TRANSIENT DYNAMIC RESPONSE ANALYSIS OF ORTHOTROPIC CIRCULAR CYLINDRICAL SHELL UNDER EXTERNAL HYDROSTATIC PRESSURE

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Following Flügge's exact derivation for the buckling of cylindrical shells, the equations of motion for transient dynamic loading of orthotropic circular cylindrical shells under external hydrostatic pressure have been formulated. The normal mode theory is used to provide transient dynamic response for the equations of motion. The effect of shell's parameters, external hydrostatic pressure and material properties on the shell response has been studied in detail. A part of tables and figures are given in this paper.

1. INTRODUCTION

Dynamic problems of circular cylindrical shells have great importance in many engineering applications, such as in the design of machines and structures. Many researchers have studied the problems. Leissa [1] has provided a comprehensive survey of free vibrations of thin and thick, isotropic and orthotropic, cylindrical shells. Dong [2] researched the free vibration of laminated shells. Warburton and Soni [3] studied the resonant response of orthotropic shells. Cederbaum and Heller [4] and Christoforou and Swanson [5] studied the response of orthotropic circular cylindrical shell subjected to impulses. Using Flügge's theory, Frederick analyzed the impact loading of a submarine isotropic shell under external hydrostatic pressure. The equations of motions in reference [6] are applicable for long or thick cylindrical circular shells.

In this paper, following Flügge's exact derivation for the buckling of cylindrical shells, the equations of motion for transient dynamic loading of orthotropic circular cylindrical shells subjected to external hydrostatic pressure have been formulated. The normal mode theory is used to provide transient dynamic response for the equations of motion. The response of displacement, strain and stress are obtained for a distributed impulse.

2. THEORETICAL ANALYSIS

Figure 1 shows an orthotropic circular cylindrical shell subjected to dynamic impulse and external hydrostatic pressure. The external hydrostatic pressure acts on the whole surface and the two ends of the shell.

With the length of shell denoted by L, the radius of the middle surface by R, the shell thickness by h, cylindrical co-ordinates (x, θ, z) are taken as shown in the figure. The external hydrostatic pressure is not shown in the figure for clearance.

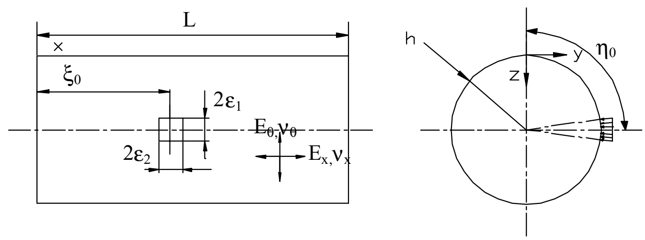

Figure 1. An orthotropic circular cylindrical shell subjected to external impact.

Suppose that the meridional and circumferential directions are principal axes of the orthotropic material. There exists the relation

$$v_x E_\theta = v_\theta E_x. \tag{1}$$

E_x, E_θ are Young's moduli and v_x, v_θ are the Poisson ratios in the axial and circumferential directions. The distributed impulse is also shown in this figure. The center of impulse is in (ξ_0, η_0).

2.1. FUNDAMENTAL EQUATIONS

The Flügge equations of shell subjected to hydrostatic pressure are written as

$$\frac{\partial N_x}{\partial x}+\frac{\partial N_{\theta x}}{R\partial\theta}-Q\left(\frac{\partial^2 u}{R\partial^2\theta}+\frac{\partial w}{\partial x}\right)-\frac{QR}{2}\frac{\partial^2 u}{\partial^2 x}+f_x(x,\theta,t)=\rho h\frac{\partial^2 u}{\partial^2 t},$$

$$\frac{\partial N_{x\theta}}{\partial x}+\frac{\partial N_\theta}{R\partial\theta}-\left(\frac{\partial M_\theta}{R^2\partial\theta}+\frac{\partial M_{x\theta}}{R\partial x}\right)-Q\left(\frac{\partial^2 v}{R\partial^2\theta}+\frac{\partial w}{\partial\theta}\right)-\frac{QR}{2}\frac{\partial^2 v}{\partial^2 x}+f_\theta(x,\theta,t)=\rho h\frac{\partial^2 v}{\partial^2 t},$$

$$\frac{\partial^2 M_x}{\partial x^2}+\frac{\partial^2 M_{x\theta}}{R\partial x\partial\theta}+\frac{\partial^2 M_{\theta x}}{R\partial x\partial\theta}+\frac{\partial^2 M_\theta}{R\partial\theta^2}+\frac{N_\theta}{R}-\frac{QR}{2}\frac{\partial^2 w}{\partial^2 x}$$

$$+Q\left(\frac{\partial u}{\partial x}-\frac{\partial v}{R\partial\theta}-\frac{\partial^2 w}{R\partial^2\theta}\right)+f_r(x,\theta,t)=\rho h\frac{\partial^2 w}{\partial^2 t}, \tag{2}$$

where ρ is the mass density of material, t the time, Q the external hydrostatic pressure, u, v, w are deflection displacements in axial, circumferential and radial directions respectively. f_x, f_θ and f_r are dynamic loading in x, θ, z directions respectively.

The component of the membrane force and the moment are given by [7]

$$N_x = D_x\left[\frac{\partial u}{\partial x}+\frac{v_\theta}{R}\left(\frac{\partial v}{\partial\theta}-w\right)\right]+\frac{K_x}{R}\frac{\partial^2 w}{\partial x^2},$$

$$N_\theta = D_\theta\left[\frac{1}{R}\left(\frac{\partial v}{\partial\theta}-w\right)+v_x\frac{\partial u}{\partial x}\right]-\frac{K_\theta}{R^2}\left(w+\frac{\partial^2 w}{\partial\theta^2}\right),$$

$$N_{x\theta} = D_{x\theta}\left[\frac{\partial u}{R\partial\theta}+\frac{\partial v}{\partial x}\right]+\frac{K_{x\theta}}{R^2}\left[\frac{\partial u}{\partial x}+\frac{\partial^2 w}{\partial x\partial\theta}\right],$$

$$N_{\theta x} = D_{\theta x}\left[\frac{\partial u}{R\partial\theta} + \frac{\partial v}{\partial x}\right] + \frac{K_{x\theta}}{R^2}\left[\frac{\partial u}{R\partial\theta} - \frac{\partial^2 w}{\partial x\partial\theta}\right],$$

$$M_x = -K_x\left[\frac{\partial^2 w}{\partial x^2} + \frac{v_\theta}{R^2}\frac{\partial^2 w}{\partial\theta^2} + \frac{1}{R}\left(\frac{\partial u}{\partial x} + \frac{v_\theta}{R}\frac{\partial v}{\partial\theta}\right)\right],$$

$$M_\theta = -K_\theta\left[\frac{1}{R}\frac{\partial^2 w}{\partial\theta^2} + v_x\frac{\partial^2 w}{\partial x^2} + \frac{w}{R^2}\right],$$

$$M_{x\theta} = -\frac{2K_{x\theta}}{R}\left(\frac{\partial^2 w}{\partial x\partial\theta} + \frac{\partial v}{\partial x}\right),$$

$$M_{\theta x} = -\frac{2K_{x\theta}}{R}\left(\frac{\partial^2 w}{\partial x\partial\theta} - \frac{1}{2R}\frac{\partial u}{\partial\theta} + \frac{1}{2}\frac{\partial v}{\partial x}\right). \tag{3}$$

The qualities D_x, D_θ, and $D_{x\theta}$ are extensional rigidities; K_x, K_θ, and $K_{x\theta}$ are flexural rigidities. They are defined as the following:

$$\begin{aligned} D_x = E_x h/(1 - v_x v_\theta), \quad D_\theta = E_\theta h/(1 - v_x v_\theta), \quad D_{x\theta} = Gh, \\ K_x = E_x h^3/12(1 - v_x v_\theta), \quad K_\theta = E_\theta h^3/(1 - v_x v_\theta), \quad K_{x\theta} = Gh^3/12, \end{aligned} \tag{4}$$

where G is the shear modulus.

Using equations (3) and (4), one can write equation (2) as

$$[L_{ij}]\{u, v, w\}^{\mathrm{T}} = \rho h\frac{\partial^2}{\partial t^2}\{u, v, w\}^{\mathrm{T}}, \quad i, j = 1, 2, 3. \tag{5}$$

$[L_{ij}]$ is linear differential operator.

2.2. DISPLACEMENT RESPONSE

Here, a simply supported shell without axial constraint at the edges is discussed. For the shell, the boundary conditions are written as

$$v = w = N_x = M_x = 0, \quad x = 0, L. \tag{6}$$

Therefore, according to normal mode method, the displacements can be taken as

$$\begin{aligned} u &= \sum_m\sum_n \bar{U}_{mn}(x,\theta)q_{mn}(t) = \sum_m\sum_n U_{mn}\cos\lambda x\cos n\theta q_{mn}(t), \\ v &= \sum_m\sum_n \bar{V}_{mn}(x,\theta)q_{mn}(t) = \sum_m\sum_n V_{mn}\sin\lambda x\sin n\theta q_{mn}(t), \\ w &= \sum_m\sum_n \bar{W}_{mn}(x,\theta)q_{mn}(t) = \sum_m\sum_n W_{mn}\sin\lambda xs\cos n\theta q_{mn}(t), \end{aligned} \tag{7}$$

where U_{mn}, V_{mn}, and W_{mn} are amplitude factors, n is the number of circumferential wave, $\lambda = m\pi/L$, m the number of axial half-waves, $q_{mn}(t)$ the generalized co-ordinate.

Substituting equation (7) into equation (5), and utilizing the orthogonality condition, yields the following:

$$\ddot{q}_{ij}(t) + \omega_{ij}^2 q_{ij}(t) = \frac{\int\int_A (f_x\cdot\bar{U}_{ij} + f_\theta\cdot\bar{V}_{ij} + f_r\cdot\bar{W}_{ij})\mathrm{d}A}{\rho h\int\int_A(\bar{U}_{ij}^2 + \bar{V}_{ij}^2 + \bar{W}_{ij}^2)\mathrm{d}A}, \tag{8}$$

where ω_{ij} is the radian frequency of vibration shape $m = i$, $n = j$ and A the integration area.

In the view of structure dynamics, the impulse is usually defined as force $\times$ time. Usually, impulse cannot be expressed by an exact mathematical equation. One can choose some impulse type, such as step impulse or triangle impulse to study the dynamic characteristics of orthotropic cylindrical shells. Therefore, we consider an impulse per unit area, $i_x(x, \theta)$, $i_\theta(x, \theta)$ and $i_r(x, \theta)$ acting on the cylinder for an infinitely short time. This kind of impulse is a simple input for theoretical analysis. The cylinder may be considered to be vibrating freely with the following initial conditions:

At $t = 0$, $u = v = w = 0$ and

$$\frac{\partial u}{\partial t} = \frac{i_x(x,\theta)}{\rho h}, \quad \frac{\partial v}{\partial t} = \frac{i_\theta(x,\theta)}{\rho h}, \quad \frac{\partial w}{\partial t} = \frac{i_r(x,\theta)}{\rho h}. \tag{9}$$

Therefore, the solution of equation (8) can be written as

$$q_{ij}(t) = \frac{\int_0^t \sin\omega_{ij}(t-\tau)\mathrm{d}\tau \int\int_A (f_x \cdot \bar{U}_{ij} + f_\theta \cdot \bar{V}_{ij} + f_r \cdot \bar{W}_{ij})\mathrm{d}A}{\omega_{ij}\rho h \int\int_A (\bar{U}_{ij}^2 + \bar{V}_{ij}^2 + \bar{W}_{ij}^2)\mathrm{d}A}. \tag{10}$$

Suppose $f_x = f_\theta = 0$, the acting area is $2\varepsilon_1 \times 2\varepsilon_2$ (as shown in Figure 1). Equation (10) can be simplified as

$$q_{ij}(t) = \frac{\int_{\eta_0-\varepsilon_1/R}^{\eta_0+\varepsilon_1/R}\int_{\xi_0-\varepsilon_2}^{\xi_0+\varepsilon_2}\int_0^t fr(x,\theta,t)\,\bar{W}_{ij}\sin\omega_{ij}(t-\tau)\mathrm{d}\tau\,\mathrm{d}x\,\mathrm{d}\theta}{\rho h\omega_{ij}\int_0^{2\pi}\int_0^l (\bar{U}_{ij}^2 + \bar{V}_{ij}^2 + \bar{W}_{ij}^2)\mathrm{d}x\,\mathrm{d}\theta}. \tag{11}$$

For $n = 0$,

$$q_{m0i}(t) = \frac{4\varepsilon_1 fr}{\rho h R\pi^2}\left(\frac{1}{m(\alpha_{m0i}^2 + 1)}\sin\frac{m\pi\xi_0}{l}\sin\frac{m\pi\varepsilon_2}{l}\right)\frac{\sin\omega_{m0i}t}{\omega_{m0i}}, \quad i = 1, 2, 3. \tag{12}$$

For $n > 0$,

$$q_{mni}(t) = \frac{8fr}{\rho h\pi^2}\left[\frac{1}{mn(\alpha_{mni}^2 + \beta_{mni}^2 + 1)}\cos n\eta_0\,\sin\frac{n\varepsilon_1}{R}\sin\frac{m\pi\xi_0}{l}\sin\frac{m\pi\varepsilon_2}{l}\right]\frac{\sin\omega_{mni}t}{\omega_{mni}},$$
$$i = 1, 2, 3. \tag{13}$$

Therefore, the displacement response of the shell can be written as

$$u(x,\theta,t) = \sum_{m=1}^{\infty}\sum_{i=1}^{3}\alpha_{m0i}q_{m0i}(t)\cos\lambda x + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{i=1}^{3}\alpha_{mni}q_{mni}(t)\cos\lambda x\cos n\theta,$$

$$v(x,\theta,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{i=1}^{3}\beta_{mni}q_{mni}(t)\sin\lambda x\sin n\theta,$$

$$w(x,\theta,t) = \sum_{m=1}^{\infty}\sum_{i=1}^{3}q_{m0i}(t)\sin\lambda x + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{i=1}^{3}q_{mni}(t)\sin\lambda x\cos n\theta. \tag{14}$$

In equation (14), α and β are mode shape coefficients, $\alpha = U_{mn}/W_{mn}, \beta = V_{mn}/W_{mn}$. The free vibration frequency and mode shape ratio can be obtained by setting the dynamic item f_x, f_θ and f_r in equation (2) to zero.

2.3. STRAIN AND STRESS RESPONSE

The strain and stress response of a point in the middle surface can be obtained by the following relations:

$$\varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_\theta = \frac{\partial v}{R\partial\theta} - \frac{w}{R}, \quad \gamma_{x\theta} = \frac{\partial u}{R\partial\theta} + \frac{\partial v}{\partial x},$$
$$\sigma_x = \frac{Ex}{1 - v_x v_\theta}(\varepsilon_x + v_\theta \varepsilon_\theta), \quad \sigma_\theta = \frac{E_\theta}{1 - v_x v_\theta}(\varepsilon_\theta + v_x \varepsilon_x), \quad \tau_{x\theta} = G\gamma_{x\theta}. \tag{15}$$

3. EXAMPLES AND DISCUSSION

In order to study the orthotropy, we choose five cases of shell properties [2]. They are listed in Table 1. Part of computation results is given. In the following computation, the acting area parameter $\varepsilon_1 = \varepsilon_2 = R/25$. The following non-dimensional parameters are used:

$$(U, V, W) = (u, v, w)\frac{48\pi E_x R^3 h}{L^3 f_r},$$
$$(\bar{\sigma}_x, \bar{\sigma}_\theta) = (\sigma_x, \sigma_\theta)/E_x,$$
$$\Omega^2 = \frac{\rho R^2 \omega^2 (1 - v_x v_\theta)}{E_x}, \tag{16}$$
$$T = t\sqrt{\frac{E_x}{\rho R^2 (1 - v_x v_\theta)}},$$

where the non-dimensionalization is made by considering the maximum values calculated from the beam theory, regarding a cylindrical shell as a beam.

3.1. SOLUTION METHOD

Because the external hydrostatic pressure exists, one should calculate the convergence load Q_{cr}. Q_{cr} can be acquired by classic method by letting dynamic items vanish in equation (2). After the convergence load has been computed, in order to study the effects of hydrostatic loading on the dynamics of the shell, we suppose some hydrostatic pressure level, that is $Q/Q_{cr} = 0$, 0·25, 0·50, 0·75, 0·98, to calculate dynamic responses of the shell. For each couple of (m, n), there are three frequencies and corresponding three mode shape coefficients. All these three frequencies and coefficients are used to calculate the response of shell.

TABLE 1

Properties of materials

Case no.	v_x	v_θ	E_x ($\times 10^{10}$) (N/m^2)	E_θ ($\times 10^{10}$) (N/m^2)	G ($\times 10^{10}$) (N/m^2)	E_θ/E_x
1	0·131926	0·012114	22·7350	2·0876	0·7958	0·0918
2	0·131926	0·04	6·8599	2·0799	0·7958	0·3032
3	0·131926	0·131926	2·0545	2·0545	0·7958	1·000
4	0·04	0·131926	2·0799	6·8599	0·7958	3·298
5	0·012114	0·131926	2·0876	22·735	0·7958	10·89

3.2. ACCURACY EXAMINATION

In order to calculate the response of the shell, the free vibration frequencies and mode shape should be acquired first. For a non-trivial solution, the determinant of their coefficients must vanish. Therefore, a frequency equation can be obtained and a standard method is used to solve that polynomial equation. An example is given in Table 2.

The response of an isotropic shell subjected to a unit impulse is acquired here and comparison is given in Table 3. PSI system is used for comparison in Table 3. Convergence of the method is also examined in this example. As can be seen in the table, the convergence is good for response analysis. In the example in Table 3, $m = 1{,}50$, $n = 0{,}49$ are used.

3.3. DISPLACEMENT RESPONSE

The response of radial displacement ($x = L/2$) is shown in Figure 2. As is shown, the existence of hydrostatic pressure can enlarge the displacement response. Three Q/Q_{cr} ratios are shown in the figure. The shape in which $Q \neq 0$ is similar to that when hydrostatic pressure is equal to zero.

The effect of L/R ratio on the response is shown in Figure 3. It is shown that the radial displacement oscillates about the static value with the period of oscillation corresponding to the mode wave that gives the lowest frequency of the shell. In the case of small value of L/R ratio, as known from the results of the free vibration, the higher modes of wave except $(m, n) = (1,1)$ appear one by one and the amplitude corresponding to them becomes predominant. Therefore, the wave pattern consisting of a combination of each mode exhibits a complicated feature for impact. This is the reason why we cannot anticipate where the maximum amplitude occurs. The response becomes smaller for lower L/R ratio when the impact vanishes. As for a longer shell, the response becomes larger, until a maximum point emerges, then it decreases while time increases.

TABLE 2

Comparison of frequency parameter Ω of unloaded ($Q = 0$) orthotropic circular cylindrical shells, for which $L/R = 2{\cdot}0$, $h/R = 0{\cdot}01$, $m = 1$

	Case no.	n					
		2	3	4	5	6	7
Warburton and Soni [3]	1	0·119875	0·085272	0·065184	0·054311	0·050975	0·054227
	2	0·207798	0·139463	0·100747	0·081915	0·079310	0·089331
	3	0·330057	0·198610	0·134684	0·113414	0·122501	0·150636
	4	0·335754	0·201822	0·148874	0·153026	0·194244	0·256190
	5	0·338104	0·210697	0·188208	0·240929	0·334672	0·453548
Present	1	0·119888	0·085277	0·065190	0·054336	0·051014	0·054286
	2	0·207822	0·139494	0·100772	0·081952	0·079335	0·089433
	3	0·330100	0·198607	0·134742	0·113515	0·122652	0·150753
	4	0·335802	0·201833	0·148975	0·153209	0·194549	0·256795
	5	0·338120	0·210729	0·188495	0·241727	0·336395	0·456826

TABLE 3

Example

	Q/Q_{cr}	$u \times 10^{12}$	$v \times 10^2$	$\varepsilon_x \times 10^5$	$\varepsilon_\theta \times 10^4$	σx	$\sigma\theta$
[7] $m = 1, 30$ $n = 0, 29$	0·00	−2·7277	2·9461	3·3372	−2·7508	−1968·3	−8908·5
	0·25	0·4485	3·5206	2·0180	−2·8794	−2558·3	−9449·1
	0·50	4·3863	4·2052	0·2079	−2·9856	−3288·7	−10 053·0
	0·75	9·5507	5·0308	−2·140	−3·0685	−4174·5	−10 597·0
Present $m = 1, 30$ $n = 0, 29$	0·00	−3·1303	2·9462	3·3345	−2·7512	−1969·7	−8910·2
	0·25	0·6743	3·5222	1·9580	−2·8802	−2579·4	−9500·4
	0·50	5·3951	4·2091	0·1483	−2·9867	−3309·9	−10 063·4
	0·75	11·1987	5·0377	−2·1666	−3·0697	−4184·6	−10 604·0
Present $m = 1, 50$ $n = 0, 49$	0·00	−3·4207	2·9495	3·2685	−2·7402	−1979·5	−8880·3
	0·25	0·1679	3·5010	1·9954	−2·8700	−2555·3	−9461·9
	0·50	4·6846	4·1628	0·28823	−2·9774	−3252·3	−10 016·3
	0·75	10·3036	4·9687	−1·9288	−3·0613	−4095·0	−10 549·0
Present $m = 1, 100$ $n = 0, 99$	0·00	−3·9468	2·9639	3·4266	−2·7365	−1922·0	−8850·1
	0·25	−0·1019	3·5109	2·0174	−2·8660	−2524·3	−9439·3
	0·50	4·7595	4·1670	0·26632	−2·9733	−3255·0	−10 004·9
	0·75	10·5577	4·9665	−2·0112	−3·0577	−4118·6	−10 546·0

Note: PSI system is used in this table

$L = 0{\cdot}6096\,\text{m}$, $R = 1{\cdot}524\,\text{m}$, $h = 3{\cdot}048 \times 10^{-2}\,\text{m}$, $v_x = v_\theta = 0{\cdot}33333$, $\xi_0 = 0{\cdot}3048\,\text{m}$, $\eta_0 = 0$, $t = 6 \times 10^{-4}\,\text{s}$, $\varepsilon_1 = \varepsilon_2 = 0.0508\,\text{m}$. Computation point: $X = \frac{1}{2}$, $\theta = 0$. Convergence load Q_{cr}: 5083.855 psi [7], this paper: 5083.849 psi.

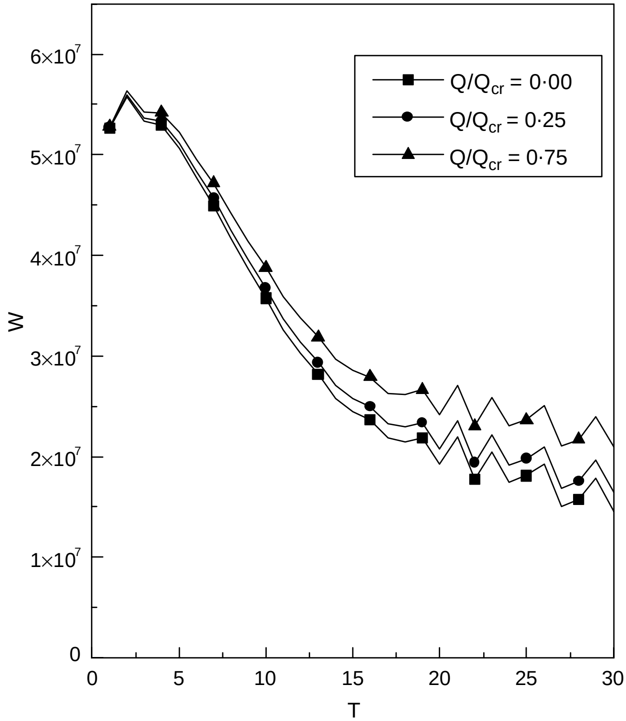

Figure 2. $W-T$ curves for $L/R = 4$, $h/R = 0{\cdot}01$, $E_\theta/E_x = 0{\cdot}0918$.

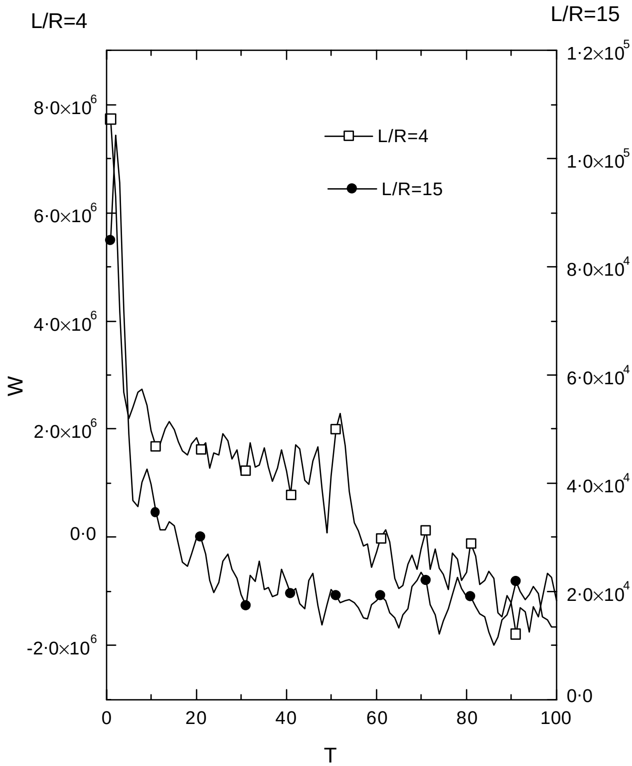

Figure 3. $W-T$ curves for $h/R=0{\cdot}01$, $E_\theta/E_x=1{\cdot}0$, $Q/Q_{cr}=0{\cdot}50$.

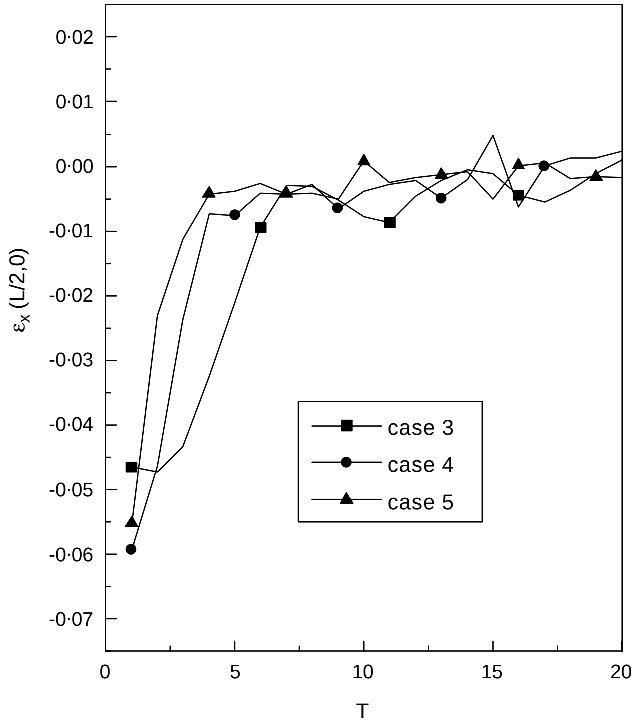

Figure 4. ε_x-T curves for $L/R=4$, $h/R=0{\cdot}01$, $E_\theta/E_x=1{\cdot}0$, $3{\cdot}298$, $10{\cdot}89$, $Q/Q_{cr}=0{\cdot}25$.

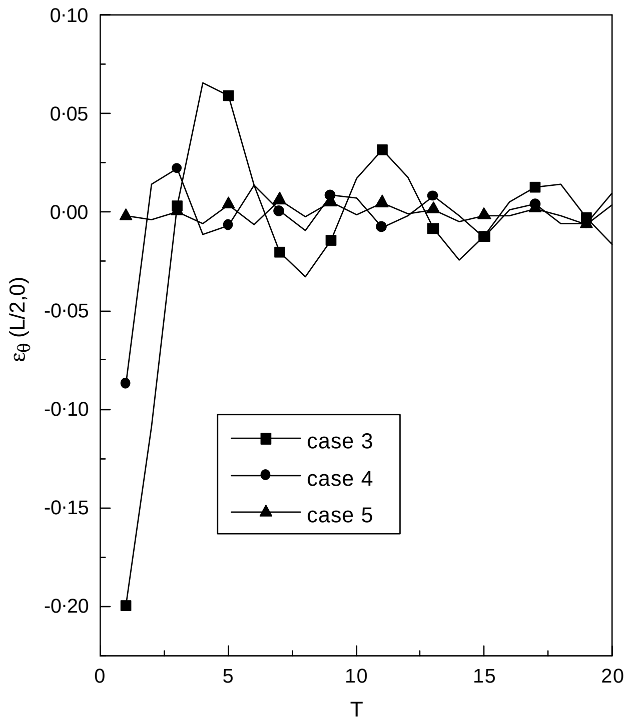

Figure 5. $\varepsilon_\theta - T$ curves for $L/R = 4$, $h/R = 0{\cdot}01$, $E_\theta/E_x = 1{\cdot}0$, $3{\cdot}298$, $10{\cdot}89$, $Q/Q_{cr} = 0{\cdot}25$.

3.4. STRESS AND STRAIN RESPONSES

The responses of stress and strain (non-dimensional) are shown in Figures 4 and 5.

As shown in Figure 4, axial strain ε_x is larger for larger E_θ/E_x ratio during initial response ($T < 7$). As for circumferential strain ε_θ response (Figure 5), it oscillates along its static value. While E_θ/E_x ratio increases, the resistance of structure to external pressure is enhanced. Therefore, the circumferential strain ε_θ decreases gradually.Stress response is shown in Figure 6.

4. CONCLUSIONS

Following Flügge's exact derivation for the buckling of cylindrical shells, the equations of motion for transient dynamic loading of orthotropic circular cylindrical shells subjected to external hydrostatic pressure have been formulated. The normal mode theory is used to provide transient dynamic response for the equations of motion. The effect of shell's parameters and material properties on the shell response has been studied.

Hydrostatic pressure can enlarge the response. The response of shorter shell is larger and an increase stage exists during initial response.

Circumferential strain ε_θ response oscillates along its static value. While E_θ/E_x ratio increases, the resistance of structure to external pressure is enhanced. Therefore, the circumferential strain ε_θ decreases gradually.

Isotropic cases can be studied by letting $E_x = E_\theta$.

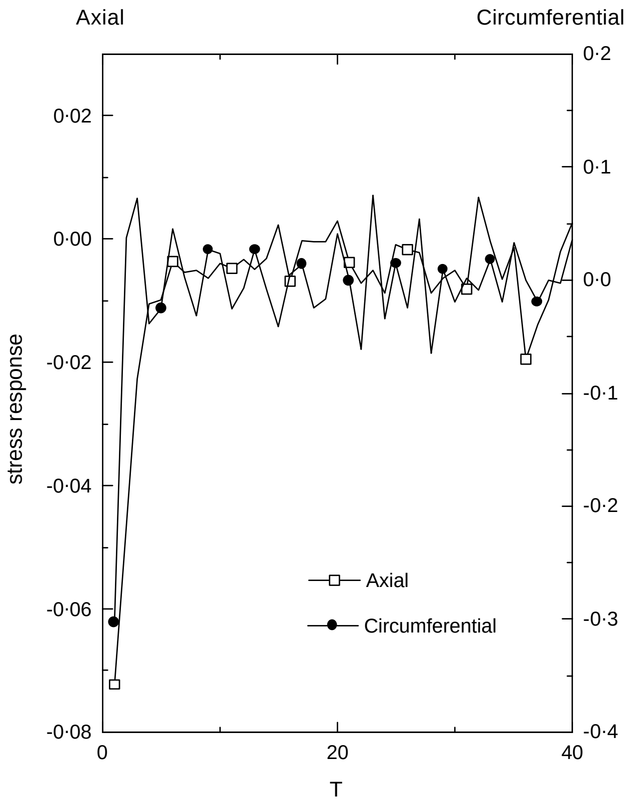

Figure 6. σ_x and σ_θ curves for $L/R = 4$, $h/R = 0{\cdot}01$, $E_\theta/E_x = 3{\cdot}298$, $Q/Q_{cr} = 0{\cdot}75$.

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